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A multi-component plasma model of a high-T_c superconductor

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Abstract. It is shown that the occurrence of negative dispersion in the ion plasma branch of a two-component plasma of electrons and ions can lead to a sharp increase in the binding energy of Cooper pairs. The analysis is generalised to a multi-component plasma with many ionic species, and the applicability of this model to high-T_c superconducting materials is discussed.

Within the framework of conventional theories the origin of the rather strong attractive interaction between the electrons on the Fermi surface forming strongly bound Cooper pairs in high-T_c superconductors has been attributed to mediation of many types of excitations. There are well known models (Ginzburg and Kirzhnits 1982) in which the attractive part of the electron–electron interaction arises from exchange of ion-acoustic plasmons. The object of this note is to consider the electron–electron interaction in a multi-component plasma consisting of electrons and ions, and show that under certain conditions affecting the dispersions of the ion plasma branches, a substantial increase in the energy gap and hence in the transition temperature can be expected.

Let us first consider a two-component plasma. In terms of its dielectric function $\varepsilon(\boldsymbol{q}, \omega)$, the electron–electron interaction matrix element between the states $|\boldsymbol{k}\rangle$ and $|\boldsymbol{k}'\rangle$ corresponding to the energies $E(\boldsymbol{k})$ and $E(\boldsymbol{k}')$ is

$$V_{kk'} = 4\pi e^2/q^2\varepsilon(\boldsymbol{q}, \omega_{kk'}) \tag{1}$$

with $\boldsymbol{q} = \boldsymbol{k} - \boldsymbol{k}'$ and $\omega_{kk'} = (E(\boldsymbol{k}) - E(\boldsymbol{k}'))/\hbar$. $\varepsilon(\boldsymbol{q}, \omega)$ can be evaluated using the hydrodynamic model for long-wave longitudinal plasma oscillations in the medium, in the form (Montgomery 1971)

$$\varepsilon(\boldsymbol{q}, \omega) = 1 - \omega_e^2/(\omega^2 - \beta_e^2 q^2) - \omega_i^2/(\omega^2 - \beta_i^2 q^2) \tag{2}$$

where $\omega_e = \surd(4\pi n_e e^2/m)$, and $\omega_i = \surd(\omega_e^2 Zm/M)$ are the electron and ion plasma frequencies, n_e is the electron density, m and M are the electron and ion masses, Z is the valency of the ion and β_e and β_i respectively are the corresponding dispersion parameters. β_e is proportional to v_F, the Fermi velocity, and hence $\beta_e^2 > 0$. β_i^2 of the ion plasma branch is governed by the non-Coulombic part of the interaction between two ions, i.e. the difference between the total and the direct Coulomb interactions between

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them. The lattice structure of the solid, as well as the transverse phonons in the system, arise from this interaction. We shall simplify the problem here, considering only the longitudinal plasmons associated with the ions. Unlike β_e^2, there is no *a priori* reason for β_i^2 to be positive. The ion plasma branch could be like the LO phonons in an ionic crystal which have negative dispersion at long wavelengths, i.e. $\beta_i^2 < 0$, if the non-Coulombic ion–ion interactions so demand. In fact, for a Coulomb lattice the LO plasma branch is known to have negative dispersion (Pines 1963). The non-Coulombic ion–ion interactions can accentuate this effect. The dominant non-Coulombic interaction would be of the van der Waals type, the strength of which is determined by the polarisability of the ions (Rehr *et al* 1975, Mahanty and Taylor 1978, Maggs and Ashcroft 1987).

There will be two coupled plasmon modes whose dispersion relations are obtained from the equation $\varepsilon(\boldsymbol{q}, \omega) = 0$. These modes (in terms of the electron and ion plasma frequency branches $\omega_e^2(\boldsymbol{q}) = \omega_e^2 + \beta_e^2 q^2$ and $\omega_i^2(\boldsymbol{q}) = \omega_i^2 + \beta_i^2 q^2$) are given by

$$\begin{aligned}\omega_P^2(\boldsymbol{q}) &= \tfrac{1}{2}\{\omega_e^2(\boldsymbol{q}) + \omega_i^2(\boldsymbol{q}) + [(\omega_e^2(\boldsymbol{q}) + \omega_i^2(\boldsymbol{q}))^2 - 4\beta_e^2\beta_i^2(q_0^2 q^2 + q^4)]^{1/2}\} \\ &\simeq \omega_e^2 + \omega_i^2 + [(\omega_e^2\beta_e^2 + \omega_i^2\beta_i^2)/(\omega_e^2 + \omega_i^2)] \qquad (3a)\end{aligned}$$

$$\begin{aligned}\omega_A^2(\boldsymbol{q}) &= \tfrac{1}{2}\{\omega_e^2(\boldsymbol{q}) + \omega_i^2(\boldsymbol{q}) - [(\omega_e^2(\boldsymbol{q}) + \omega_i^2(\boldsymbol{q}))^2 - 4\beta_e^2\beta_i^2(q_0^2 q^2 + q^4)]^{1/2}\} \\ &\simeq v_S^2 q^2 \qquad (3b)\end{aligned}$$

where the second equality in each case is the form in the long-wave limit, and

$$q_0^2 = \omega_e^2/\beta_e^2 + \omega_i^2/\beta_i^2 \qquad v_S^2 = \beta_e^2\beta_i^2 q_0^2/(\omega_e^2 + \omega_i^2). \qquad (4)$$

v_S is the sound velocity corresponding to the ion-acoustic branch. When β_i^2 is negative, for stability of the solid v_S must be real, or $(\beta_i^2 q_0^2)$ must be positive. This condition constrains the negative β_i^2 to satisfy the inequality $|\beta_i^2| < \beta_e^2\omega_i^2/\omega_e^2$. For $|\beta_i^2| = 0$, we get the well known result for the sound velocity with an undispersed ion plasma branch (de Gennes 1966, Kubo and Nagamiya 1969), $v_0^2 \simeq \beta_e^2\omega_i^2/\omega_e^2 \simeq v_F^2(Zm/M)$.

The effect of negative dispersion in the ion plasma branch on electron–electron interaction is substantial, and this aspect will be discussed in detail hereafter. Equation (1) can be written as

$$V_{kk'} = (4\pi e^2/q^2)(\omega_{kk'}^2 - \beta_e^2 q^2)(\omega_{kk'}^2 + |\beta_i^2| q^2)/(\omega_{kk'}^2 - \omega_P^2(\boldsymbol{q}))(\omega_{kk'}^2 - \omega_A^2(\boldsymbol{q})). \qquad (5)$$

Since ω_A is of the order of the Debye frequency ω_D, the interaction is attractive in the region $|\omega_{kk'}| < \omega_D$, as in the conventional theory. The zero-temperature gap equation is (Schrieffer 1964)

$$\Delta(\boldsymbol{k}) = -\tfrac{1}{2}\sum_{k'} \frac{V_{kk'}\Delta(\boldsymbol{k}')}{\sqrt{(E^2(\boldsymbol{k}') + \Delta^2(\boldsymbol{k}'))}} \qquad E(\boldsymbol{k}) = \hbar^2 k^2/2m - E_F. \qquad (6)$$

E_F is the Fermi energy. Taking the electron pair on the Fermi surface, in the range $|\omega_{kk'}| < \omega_D$, $V_{kk'}$ has the approximate form (for $q_0^2 < 0$),

$$\begin{aligned} V_{kk'} &\simeq \frac{4\pi e^2}{q^2}\left(\frac{-\beta_e^2|\beta_i^2| q^4}{\omega_P^2(\boldsymbol{q})\omega_A^2(\boldsymbol{q})}\right) \simeq \frac{4\pi e^2}{q^2 - \kappa_0^2} \\ \kappa_0^2 &\equiv |q_0^2| = \omega_i^2/|\beta_i^2| - \kappa_{TF}^2 \qquad \kappa_{TF} = \omega_e/\beta_e. \qquad (7)\end{aligned}$$

$(\kappa_{TF})^{-1}$ is the Thomas–Fermi screening length. In real space the pair potential will be oscillatory for $q_0^2 < 0$. This feature is reminiscent of a model due to Kohn and Luttinger (1965), where the formation of the Cooper pair occurs through the attractive regions of the pair potential obtained through a dielectric function that has Friedel oscillations due to the sharpness of the Fermi surface. The oscillations obtained from (7), however, are much stronger than in the latter situation.

At this stage $V_{kk'}$ of (7) can be compared with that in other models, such as those involving the electrons being coupled to LO phonons (Tachiki and Takahashi 1988, Dolgov *et al* 1987). In the latter models also, there could be considerable enhancement in $V_{kk'}$. But the mechanism proposed in this note is simpler in concept, and depends only on the details of the inter-ionic forces.

The gap equation can be simplified, by writing the $\boldsymbol{k}'$-integral as integrals over q and E as is customary in such problems (Scalapino 1969), to the form

$$1 = -\frac{N(0)\pi e^2}{k_F^2} I \int_{-\hbar\omega_D}^{\hbar\omega_D} \frac{dE}{\sqrt{(E^2 + \Delta^2)}} \tag{8}$$

with

$$I = \int_0^{2k_F} \frac{q\,dq}{q^2 - \kappa_0^2} = \begin{cases} \frac{1}{2}\ln(1 - 4k_F^2/\kappa_0^2) & \text{for } \kappa_0 > 2k_F \\ \frac{1}{2}\ln(4k_F^2/\kappa_0^2 - 1) & \text{for } \kappa_0 < 2k_F \end{cases} \tag{9}$$

and $N(0)$ is the density of states at the Fermi surface. Here Δ is taken as independent of $\boldsymbol{k}$. The form of I in (9) is obtained from the small-q form of $\varepsilon(\boldsymbol{q}, 0)$, and effects such as local field corrections at large q are not taken into consideration here. We believe that this approximation will not substantially alter the following deductions.

For I to be negative, which is the condition for the existence of the non-trivial solution of the gap equation, κ_0 must satisfy the inequality $\kappa_0 > \sqrt{2}k_F$, and in this region there is a logarithmic singularity at $\kappa_0 = 2k_F$. The solution of the gap equation then becomes

$$\Delta = \hbar\omega_D / \sinh(k_F^2 / 2N(0)\pi e^2 |I|). \tag{10}$$

Here the value $\pi e^2 |I| / k_F^2$ is like the coupling constant of BCS theory, and (10) reduces to the BCS result when this value is small. However, depending on the dispersion of the ion plasma branch, $|I|$ can be large and thus the possibility of Δ being of the order of $\hbar\omega_D$ emerges.

Equation (4) for v_S can be written as (with $\omega_i \ll \omega_e$)

$$v_S^2 = v_0^2 - |\beta_i^2| \tag{11}$$

where v_0 has been defined earlier. The condition for lattice stability is $0 < v_S^2 < v_0^2$, and this restricts the value of $|\beta_i^2|$.

In terms of the parameter $x = v_S^2/v_0^2$, I as a function of x is given in figure 1. It is clear that a non-trivial solution for Δ in (10) can exist only for values of x in the range $x_0 < x < 1$, where $x_0 = 2k_F^2/(2k_F^2 + \kappa_{TF}^2)$. From (10), $\Delta/\hbar\omega_D$ as a function of x is given in figure 2. For these calculations we have taken $r_S = 10$ ($r_S \equiv (3/4\pi n_0)^{1/3}$) corresponding to the electron density $\simeq 1.6 \times 10^{21}$ cm^{-3} (Onyszkiewicz *et al* 1988).

Although the value of Δ is unrealistic near the singularity of I, it is possible to get values of Δ of the order of $\hbar\omega_D$ in a range of values of x, and hence of $|\beta_i^2|$.

Since high-T_c superconductors contain many ionic species, the above analysis in terms of a two-component plasma model must be generalised to a multi-component plasma model to be applicable to real systems. This can be done as follows.

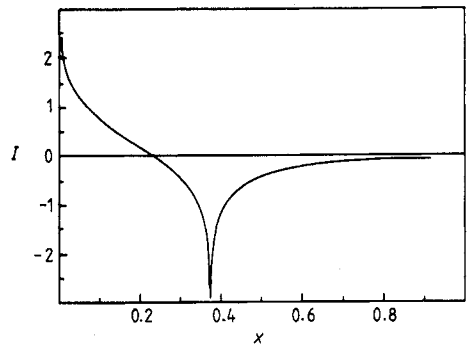

Figure 1. I from (9) as a function of $x = v^2_S/v^2_0$ for $r_s = 10$.

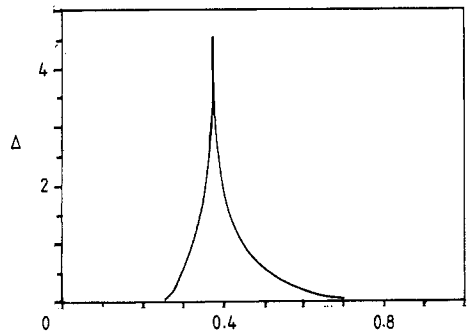

Figure 2. Δ (in units of $(\hbar\omega_D)$) as a function of $x = v^2_S/v^2_0$ for $r_s = 10$.

The dielectric function of a multi-component plasma is

$$\varepsilon(\boldsymbol{q}, \omega) = 1 - \sum_{\nu=1}^{p} \frac{\omega_\nu^2}{\omega^2 - \beta_\nu^2 q^2}. \tag{12}$$

where $\nu = 1$ is the electron plasma, and $\nu = 2, 3, \ldots, p$ represent the various ionic species. ω_ν and β_ν are respectively the plasma frequency and the dispersion parameter of the νth ion species. The ion-acoustic branch can be obtained from the equation $\varepsilon(\boldsymbol{q}, \omega) = 0$ in the form

$$\omega_A^2(\boldsymbol{q}) \simeq v_S^2 q^2 \tag{13}$$

$$v_S^2 = \sum_{\nu=1}^{p} \beta_\nu^2 - \left(\sum_{\nu=1}^{p} \omega_\nu^2 \beta_\nu^2\right) \Big/ \sum_{\nu=1}^{p} \omega_\nu^2 \simeq \sum_{\nu=2}^{p} \beta_\nu^2 + \beta_1^2 \left(\sum_{\nu=2}^{p} \omega_\nu^2\right) \Big/ \omega_1^2$$

$$= v_0^2 + \sum_{\nu=2}^{p} \beta_\nu^2 \tag{14}$$

with

$$v_0^2 = \frac{\beta_1^2}{\omega_1^2} \sum_{\nu=2}^{p} \omega_\nu^2 = \left(\sum_{\nu=2}^{p} \omega_\nu^2\right) \Big/ \kappa_{TF}^2. \tag{15}$$

The last form for v_S^2 in (14) follows from the fact that $\omega_1^2 \gg \omega_\nu^2$, for $\nu = 2, \ldots, p$. Equation (15) can be written as

$$v_0^2 = \beta_1^2 (m/\bar{M}) \qquad \frac{1}{\bar{M}} = \left(\sum_{\nu=2}^{p} n_\nu Z_\nu \frac{Z_\nu}{M_\nu}\right) \Big/ \sum_{\nu=2}^{p} n_\nu Z_\nu \tag{16}$$

where Z_ν and M_ν are the valency and mass of the νth species of ion, n_ν is its density, and charge neutrality of the system implies

$$n_1 = \sum_{\nu=2}^{p} n_\nu Z_\nu.$$

We also have

$$\frac{1}{q^2 \varepsilon(\boldsymbol{q}, 0)} = \frac{1}{q^2 + q_0^2} \qquad q_0^2 = \sum_{\nu=1}^{p} \frac{\omega_\nu^2}{\beta_\nu^2}. \tag{17}$$

If some of the ion plasma branches have negative dispersion, i.e. $\beta_\nu^2 < 0$ for some ν, and those $|\beta_\nu^2|$ are small, the possibility arises of q_0^2 becoming negative. One can then define

$$\kappa_0^2 \equiv |q_0^2| = \sum_\mu \frac{\omega_\mu^2}{|\beta_\mu^2|} - \left(\kappa_{\mathrm{TF}}^2 + \sum_\nu \frac{\omega_\nu^2}{\beta_\nu^2}\right) \tag{18}$$

where the branches with negative dispersion have been separated into the summation over μ. Also, for lattice stability, $v_S^2 > 0$ and this constraints the β_ν^2 to satisfy the inequality

$$-v_0^2 < \sum_{\nu=2}^{p} \beta_\nu^2. \tag{19}$$

The sound velocity v_S in the medium is less than v_0 if the sum in (19) is negative.

The subsequent analysis is identical to that given in equations (6) to (10) with κ_0 defined as in (18). The parameter x, which contains all the information about dispersion of the ion plasma branch in the two-component case, is not useful in the multi-component system, since it will not identify which ion plasma branches have negative dispersion. But Δ as a function of κ_0 will be exactly as in the two-component case. Since the mass dependence of v_0 is averaged over the ions in the manner shown in (16), the isotope effect associated with any particular species will be small, if not negligible.

Although the oxide superconductors are structurally anisotropic and are considered as layered materials, several thermal and electrical measurements suggest that the superconductivity is bulk (three-dimensional) in nature (Inderhees *et al* 1988, Goldenfeld *et al* 1988). Recently Gersten (1988) has made an analysis using a hydrodynamical approach similar to ours in a two-dimensional model. That would certainly be necessary if the three-dimensional models prove inadequate.

The realisation of high-T_c superconductivity in our model depends on the particular form of $\varepsilon(\boldsymbol{q}, 0)$ arising out of negative (and small) dispersion of the ion plasma branch in the two-component case, and a net negative dispersion in those branches in the multi-component case. Other processes contributing to $\varepsilon(\boldsymbol{q}, 0)$ which give it a similar structure would obviously contribute to high-T_c superconductivity. *Ab initio* evaluation of the dispersion of an individual ion plasma branch in a typical high-T_c superconducting material requires detailed information on the non-Coulombic part of the ion–ion interaction in the sublattice of that ionic species. This is not attempted in this paper.

A direct consequence of this negative dispersion in the ion plasma branches would be a lowering of the longitudinal sound velocity from v_0 to v_S. Such a reduction of sound velocity was considered earlier by Kulik (1965) as a criterion for superconductivity. From a recent measurement (Jericho *et al* 1988), sound velocity in $YBa_2Cu_3O_{7-\delta}$ has been found to be very low compared with that in metallic superconductors. The electron density in $YBa_2Cu_3O_{7-\delta}$ is $\simeq$(1–2) $\times 10^{21}$ cm^{-3}, corresponding to $r_s \simeq 10$. From this we obtain $v_0 \simeq 2 \times 10^6$ cm s^{-1} (by taking M as oxygen mass and m as the bare electron mass). The observed sound velocity $v_S \simeq 2.1 \times 10^5$ cm s^{-1}. This reduction of the sound velocity is perhaps an indication of net negative dispersion in the ion plasma branches.

Note added in proof. Our object in this paper was to examine the crucial role of possible negative dispersion in the ion plasma branches on the screening of electron–electron interaction. However, a number of papers have appeared on the bulk and layered (electron) plasmons believed to be responsible for the high-T_c superconductivity. See, for example, Kresin (1987), Ruvalds (1987), Ashkenazi *et al* (1987), Griffin (1988) and others. Recently Kresin and Morawitz (1989) have considered a combined phonon–plasmon mechanism in the Eliasberg formalism.

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